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2401. Consider whether the statistics are measures of central tendency or spread.

2402. Since you are working near the origin, i.e. with small x values, use a small-angle approximation $\sin x \approx x$. Check error bounds at the end, to show that your root is correct to 1dp.

2403. (a) Use NIII.
(b) Use *suvat*.
(c) Add the accelerations from (b), and then use a single *suvat*.

2404. The point lies on the x axis, and is stationary. The question is the nature of the stationary point. You need to consider the fact that the curve is a cubic.

2405. Enact the differential operator, using the chain or quotient rule.

2406. Let the indefinite integral of y be $\mathrm{F}(x)$. This will allow you to perform the integrals and check the algebra.

2407. Consider the order of $\mathrm{f}(x)\,\mathrm{g}(x)$, its sign, and the multiplicity of the roots.

2408. Go through the motions of solving to find A and B, and find a contradiction.

2409. (a) Substitute t and T.
(b) Rearrange the first equation for A.
(c) Divide the equations and solve a quadratic in 2^λ.
(d) Take the limit by setting the exponential decay term to zero.

2410. Easiest is to rearrange to $y \sin u = 0$, and then differentiate implicitly with respect to y.

2411. (a) Sketch $y = x^2 - 1$ first, and then consider the effect of applying the mod function to the y values.
(b) Both intersections appear on the same part of the graph (the bit that has been reflected). So, you only need to solve one equation.

2412. You don't need to use the fact that the vertices form a cyclic quadrilateral. Easier is simply to find the midpoints of the diagonals, and show that they coincide.

2413. Simplify the LHS using the binomial expansion. Then solve the boundary equation. Sketch a graph if you need to.

2414. The point of this question is that simultaneous solving requires *both* conditions to hold: note that the logical negation of "A and B are true" is not "A and B are false." Rather, it is "At least one of A and B is false."

2415. Find a counterexample: two prime numbers whose mean is prime.

2416. Use the compound-angle formula

$$\sin(\theta - \phi) \equiv \sin\theta\cos\phi - \cos\theta\sin\phi.$$

Quote the exact values for 45° and 30°.

2417. Use an input transformation to reflect the parabola in the y axis. Note that, if the new parabola is a translation of P in the x direction, then it is also a reflection of P in the x direction.

2418. Write the numerator in terms of the denominator, i.e. $4x = 2(2x-1)+2$. Then split the fraction up.

2419. A number line may help. In both, the limits are 0 and 1. In (b), the symbol is set subtraction.

2420. The *third difference* is the "difference between the differences between the differences between terms." Visually, this is

1		8		27		64	Terms
	7		19		37		First Diff.
		12		18			Second Diff.
			6				Third Diff.

Consider the polynomial order (highest power) of the second differences, first differences, and thus original terms.

2421. Calculate a definite integral.

2422. Express e as 10^k, and then manipulate $y = e^x$ into the form $y = 10^{\mathrm{f}(x)}$.

2423. Solve $\mathrm{f}(x) = 1$; it's a quadratic. Then differentiate either by the product rule or by first multiplying out. Then evaluate f' at the roots of $\mathrm{f}(x) = 1$.

2424. Form an equation linking m and n, and so express $m^2 + n^2$ in terms of one variable only.

2425. Newton-Raphson tends to be easiest. You might want to do some rearranging first: the equation can be manipulated to a quartic polynomial.

2426. This is a standard result known as integration by inspection. Inspections are justified (once you know the result) by running the process in reverse: differentiating to get the integrand.

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2427. (a) What would need to be true about the string for the masses to have *different* accelerations?

  (b) Resolve along the string.

2428. Use log rules.

2429. The volume formula is $V = \frac{1}{3}\pi r^2 h$. Differentiate this implicitly with respect to t, then substitute in the various quantities.

2430. You need to show that the quadratic $5x^2 - 3x - 1$ is strictly greater than zero for all $x \in [1, \infty)$.

2431. Express the fraction as an integer plus a proper fraction: begin by writing the top in terms of the bottom.

2432. Show that $\operatorname{cosec}\theta$ has the same value whether the interior angle chosen is acute or obtuse. So, call the angle at the right-hand vertex θ. Split this this in two, and think of the rhombus as four right-angled triangles. You'll need a double-angle formula.

2433. Both cubics have $x = 0$ as a root. The question is whether there are any possibilities other than $x = 0$.

2434. A logarithm is unaffected when raising both base and input to the same power.

2435. (a) Use NIII. If the boy exerts a force of 400 N on the sledge, then the sledge must exert a force of 400 N on the boy.

  (b) Resolve horizontally for the sledge.

2436. Sketch the situation carefully: once you have found the relationship between the boundary line and the circle, no calculation is required.

2437. The even and odd cases are different.

2438. You can prove this by construction, which means laying out the means of finding the circle through the vertices, and establishing that there is only way of doing so.

2439. (a) For the LHS, write the first ten terms longhand and add them.

  (b) For the RHS, quote the trapezium rule formula, and calculate it.

2440. Begin by finding the sum of the integers from 1 to 100 which *are* multiples of 5. You might want to use the fact that the sum of the first n integers is $S_n = \frac{1}{2}n(n+1)$.

2441. Differentiate by the chain rule to find the gradient at $x = \frac{\pi}{3}$. Find the y coordinate, and substitute into $y - y_1 = m(x - x_1)$.

2442. (a) Average the position vectors.

  (b) You'll need the fact that $\overrightarrow{AP} = \mathbf{p} - \mathbf{a}$.

  (c) You should find that X and Y coincide when λ and μ are $\frac{2}{3}$. This means that the *centroid* (the point of intersection) divides both AP and BQ in the same ratio $2 : 1$.

  (d) To complete the proof, extend the argument by symmetry to CR.

2443. The basic sinusoids $\sin(x)$ and $\cos(x)$ have period 2π. Consider e.g. $\sin 5x$ as an input-transformed sine wave, whose period is $\frac{2}{5}\pi$.

2444. Use the quotient rule.

2445. (a) Take the RHS and simplify to get the LHS.

  (b) Use integration by inspection, here the reverse chain rule.

2446. Use a log law to express the LHS as a single ln. Then undo the ln by exponentiating both sides. Then use a double-angle formula. There is one root in the given domain.

2447. Differentiate, and then find the equation of a generic tangent in the form $y = mx + c$, where both m and c depend on a. Then find the points at which this line crosses the axes, and calculate the area of $\triangle AOB$.

2448. This is mostly valid, but one special case has been ignored.

2449. Find the first derivative at $x = \frac{\pi}{4}$. If you try to find its negative reciprocal, something will go awry. But this isn't an error: think about the graph and interpret the result.

2450. (a) Use *suvat*.

  (b) Use NII for the person.

  (c) Use NII and *suvat* for the Earth.

  (d) Use NII for the Earth, remembering that weight (gravitational attraction) also comes as a Newton III pair.

  (e) Use *suvat*.

2451. You might first consider how $y = x$ relates to (a) $|y| = x$ and (b) $y = |x|$. All three pass through the origin. You can then translate them by vector $p\mathbf{i} + q\mathbf{j}$.

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2452. The function f is increasing, so the first derivative is positive everywhere. Consider what this means in visual/graphical terms, looking for intersections with (a) $y = 0$, (b) $y = x$, (c) $y = -x$.

2453. In a large population, the probability that any member lies above the upper quartile is 0.25.

2454. Consider the effect of the given transformation on the initial position vector $5\mathbf{i}$ and on the direction vector $\mathbf{i} + 3\mathbf{j}$.

2455. Find a counterexample. The statement is true for polynomials, so you need to look for something else. Note that nothing in the question says you can't define a graph *piecewise*, i.e. differently in different domains.

2456. Solve $1170x^2 - 389x - 165 = 0$ first using the quadratic formula, and then reverse engineer the factorisation.

2457. (a) Use Pythagoras.
   (b) Calculate gradients.
   (c) Consider that $\mathbf{p}$ is the standard position vector on the unit circle, at angle θ.
   (d) Solve a pair of simultaneous equations for $\mathbf{i}, \mathbf{j}$.

2458. Divide top and bottom by x^2 before taking the limit.

2459. The question is, in each case, whether the graph $y = \cos x$ has one of the coordinate axes as a line of symmetry.

2460. Prove this by contradiction. Consider the cases $p = 3k$ and $p = 3k + 1$ for $k \in \mathbb{N}$, and show that each leads to a contradiction.

2461. This is best considered graphically.

2462. In each, find the range R of the denominator first. Then reciprocate, considering whether R contains 0 or not.

2463. This is a quadratic in $x^{\frac{3}{2}}$.

2464. In each case, fill the first gap, then the second, then the third.

2465. Integrate by the reverse chain rule, and use the exact values of sin.

2466. This is true. Consider the two linear equations $ax + by = c$ and $dx + ey = f$ as straight lines on a set of (x, y) axes.

2467. Use Pythagoras to find the radius of the circle in terms of l.

2468. Differentiate by the quotient rule, and then set the numerator to zero.

2469. Use the binomial expansion. Note that half of the terms will cancel, and don't need to be worked out explicitly.

2470. The student is correct. Consider NIII.

2471. Differentiate and solve $\frac{dy}{dx} > 0$ for k.

2472. Consider the three transformations: input scaling, output scaling, translation. Only two affect areas.

2473. This can be done by drawing two individual force diagrams, or more efficiently by considering the equation of motion along the string.

2474. Explain why $\arccos x = \theta$, where θ is the standard angle on the unit circle.

2475. Only (b) has insufficient information. In (a) and (c), proceed algebraically from the equation given.

2476. Consider the fact that the curves are reflections of each other in $y = x$.

2477. (a) Differentiate.
   (b) Solve for intersections between the tangent line and the curve.
   (c) Use the factor theorem. You already know one of the roots of the cubic.

2478. (a) Sketch the distribution.
   (b) The inequality $X^2 < k^2$ is a restriction of the possibility space. Again, a sketch will help.

2479. The functions are quadratic, so their derivatives are linear. Show that the derivatives must be the same for all x, then integrate.

2480. Consider whether the mod function is affecting the inputs or the outputs.

2481. The restrictions are only very small: most values of p and q, but not quite all, do give a stationary point at $x = 0$. Go about finding stationary points in the usual way, and substitute in the information given.

2482. Use the first Pythagorean trig identity to turn this into a quadratic in $\sin x$.

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2483. Find the relevant horizontal speed. Then, use a vertical *suvat* to find the time of flight for both. Substitute this into the horizontal.

2484. All orders of $\{x_1, x_2, x_3, x_4\}$ are equally likely, and this is exactly one of them.

2485. If in doubt, sketch $y = \mathrm{g}(x)$.

2486. (a) The ordinal formula is the nth term formula. Quote a standard GP result.
(b) Set up an equation in n and solve.

2487. (a) Use $x = e^{\ln x}$.
(b) Use index laws and the chain rule.

2488. In each case, the new graph contains the old one and some more points.

2489. Use the first Pythagorean trig identity to form a quadratic in $\sin x$.

2490. Write the information out longhand, and use the chain rule in the form $\frac{dz}{dx} = \frac{dz}{dy} \times \frac{dy}{dx}$.

2491. Calculate the two integrals. This will produce a quartic in k. Use your calculator's polynomial solver, or a numerical method, to solve for k.

2492. Find an expression for the area A in terms of t. Simplify it with a double-angle formula, and set its derivative to 0.

2493. Ask: "In the first equation, what do I replace x with?" and "In the first equation, what do I add to the outputs?" These give, respectively, the x and y transformations.

2494. With the minimum possible force, the sledge moves at constant velocity, so $a = 0$, and friction is at F_{max}.

Draw a force diagram, including the reaction force. Calculate the reaction using vertical equilibrium. This gives the frictional force. Then consider the horizontal forces.

2495. There are three ways in which a set of three lines can be non-concurrent: none parallel, two parallel, or three parallel.

2496. The identity is automatically true for $x \geq 0$. So, consider the case $x < 0$.

2497. "Verify" means "Check that the given solution works." Multiply the equation of the curve out to simplify it. Then differentiate to find $\frac{dy}{dx}$ and substitute this and y into the DE.

2498. Find a counterexample: a quartic equation (left in factorised form) with precisely three roots.

2499. Let $P = x^3 + y^3$. Rearrange $x^2 + y^2 = 1$ for y, and then substitute into $P = x^3 + y^3$. Then find $\frac{dP}{dt}$ and set it to zero.

2500. These are instances of the chain rule.

——— End of 25th Hundred ———

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